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A Computer Algorithm for Simulating the Spread of Wildland Fire Perimeters for Heterogeneous Fuel and Meteorological Conditions

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Abstract. This work describes a computer based technique for simulating the spread of wildland fire for heterogeneous fuel and meteorological conditions. The mathematical model is in the form of a pair of partial differential equations, and can model fuels whose fire perimeter for homogeneous conditions is any given shape, such as ellipses, double ellipses, lemniscates etc. Provided the fire does not attempt to burn into an already burnt out region, then the differential equations are easily solved and a simple method of solution is presented. To identify regions that are internal to the fire perimeter an algorithm that uses the turning number of a point in the plane relative to the fire perimeter is used. The algorithm is found to be reliable, and allows for the simulation of highly complex fire scenarios in a reasonable time. Example simulations are presented that involve variations in fuel, barriers, wind direction changes and multiple fires.

Keywords: Fire spread; Computer simulation; Differential equations

Introduction

The ability to simulate and hence predict the spread of a 2 dimensional fire for heterogeneous conditions is a difficult problem and valuable tool in fire management. Due to the diversity of possible fuel types, the complexity of the physical/chemical processes and computational problems in the simulation process there has been only limited success in the solution of the problem, and it may some time before it is fully solved although inroads have been made by a number of workers. The most successful techniques to date, (in the authors' opinion), are those that represent the fire perimeter as a closed curve that expands in time in a manner dictated by the mathematical model used, Wallace, (1993), Knight and Coleman, (1993), Richards, (1990), Roberts, (1989), Anderson et al., (1982). Curve expansion techniques are not without their problems, a particular difficulty being the detection of whether a region has been burnt by the fire, so that a separate part of the fire may not burn into that region at a later time. Recently, techniques for the detection of burnt regions have been proposed by Wallace, (1993), and Knight and Coleman, (1993).

A simulation procedure consists of two main components, a mathematical model to describe the phenomena and a solution procedure to extract the predictions of the model. The model used here is the differential equation model proposed by Richards, (1994), and the purpose of this work is to demonstrate how the differential equations can be solved numerically, and how burnt regions can be detected. The algorithm for the detection of burnt areas is based on identifying whether a region is internal to the curve by calculating its turning number, the algorithm is found to be reliable and allows the simulation of complex fire scenarios.

Mathematical Model

The mathematical model used is the differential equation method proposed by Richards, (1994). The fire perimeter at time t is represented parametrically in s as a closed curve $(x(s,t), y(s,t))$, $0 \leq s \leq 2\pi$, where the time derivatives $x_t(s,t)$ and $y_t(s,t)$ satisfy the partial differential equations.

$$x_t(s,t) = X(\phi(s,t) - \theta(s,t),s,t)\cos\theta(s,t) - Y(\phi(s,t) - \theta(s,t),s,t)\sin\theta(s,t) \quad (1)$$

$$y_t(s,t) = X(\phi(s,t) - \theta(s,t),s,t)\sin\theta(s,t) + Y(\phi(s,t) - \theta(s,t),s,t)\cos\theta(s,t) \quad (2)$$

where $\phi(s,t)$ is the angle of the normal vector and $\theta(s,t)$ is the angle of the wind direction to the x-axis, both at $(x(s,t),y(s,t))$ on the perimeter. The basic premise of the model is that for a given set of fuel conditions and wind speed, the rate of spread at any point on the fire perimeter is a function of the angle between the normal vector and the wind direction, with the functional relationship being controlled by the functions X and Y.

For each point $(x(s,t),y(s,t))$ on the perimeter the functions X and Y are defined by the fuel and wind speed conditions instantaneously occurring at that point. The functions are such that if fuel and wind speed conditions were homogeneous and equal to those at $(x(s,t),y(s,t))$, then the curve $(X(\phi,s,t),Y(\phi,s,t))$, $0\leq\phi\leq2\pi$, will define the fire perimeter shape after a unit of time for a point source ignition at the origin and a wind direction of $\theta = 0$. A mathematical form for $X(\phi,s,t)$ and $Y(\phi,s,t)$ has been proposed by Richards, (1994), that allows for most of the observed perimeter shapes to be represented.

Together with the initial conditions

$$x(s,0) = \hat{x}(s) \quad (3)$$
$$y(s,0) = \hat{y}(s) \quad (4)$$

where $(\hat{x}(s),\hat{y}(s))$ parametrically represents the fire at time t=0, eqns. (1,2) can in principle be solved to trace the progress of the fire.

Basic Solution Procedure

Eulers Method

Despite the notational complexity of eqns. (1,2) they are merely expressing that the time derivatives $x_t(s,t)$ and $y_t(s,t)$ are functions of the fuel and meteorological conditions at $(x(s,t),y(s,t))$ and the spatial derivatives $x_s(s,t)$ and $y_s(s,t)$ that define the orientation of the curve to the wind direction. The differential equations can therefore be written as:

$$x_t(s,t) = F(s,t,x_s(s,t),y_s(s,t)) \quad (5)$$
$$y_t(s,t) = G(s,t,x_s(s,t),y_s(s,t)) \quad (6)$$

where F and G are the R.H.S. functions of eqns. (1,2).

Eqns. (5,6) are one of the simplest types of time dependent partial differential equations and there is a wealth of numerical methods for their solution, provided the functions F and G can be evaluated. Further work is required to determine the most effective numerical solution technique. For the purposes of illustration Euler's Method was used and was found to be reasonably effective.

If Euler's Method is used, then starting with the given fire perimeter at t=0 the perimeters are approximated at intervals of time Δt apart, i.e. at times $j\Delta t, j$ = 1,2,3... . At time $j\Delta t$ the perimeter is discretised as the set of n points $(x(i\Delta s,j\Delta t),y(i\Delta s,j\Delta t))$, i = 0... n - 1, where $\Delta s = 2\pi/n$. The notation is used that $(x_{i,j},y_{i,j})$ represents the point $(x(i\Delta s,j\Delta t),y(i\Delta s,j\Delta t))$, so that the set of n points $(x_{i,j},y_{i,j})$, i = 0... n -1, approximate the perimeter at time $j\Delta t$.

If at some point the time derivatives $x_t(i\Delta s,j\Delta t)$ and $y_t(i\Delta s,j\Delta t)$ are approximated by the forward difference approximations $(x_{i,j+1}-x_{i,j})/\Delta t$ and $(y_{i,j+1}-y_{i,j})/\Delta t$ respectively, and the spatial derivatives $x_s(i\Delta s,j\Delta t)$ and $y_s(i\Delta s,j\Delta t)$ are approximated using the central difference approximations $(x_{i+1,j}-x_{i-1,j})/2\Delta s$ and $(y_{i+1,j}-y_{i-1,j})/2\Delta s$ respectively, then substituting these into eqns. (5,6) gives that

$$x_{i,j+1} = x_{i,j}+\Delta tF(s,t,(x_{i+1,j}-x_{i-1,j})/2\Delta s,(y_{i+1,j}-y_{i-1,j})/2\Delta s) \quad (7)$$

$$y_{i,j+1} = y_{i,j}+\Delta tG(s,t,(x_{i+1,j}-x_{i-1,j})/2\Delta s,(y_{i+1,j}-y_{i-1,j})/2\Delta s) \quad (8)$$

so that the approximation to the perimeter at time $(j+1)\Delta t$ is expressed as a recurrence relation in terms of the perimeter at time $j\Delta t$. The initial conditions are taken from the initial conditions of the differential equations, eqns (3,4) and are given by

$$x_{i,0} = \hat{x}(i\Delta s) \quad (9)$$
$$y_{i,0} = \hat{y}(i\Delta s) \quad (10)$$

By starting at time t = 0 then the eqns. (7,8) together with the initial conditions allow the progress of the fire to be traced at intervals of time Δt apart. Smaller values of Δt and Δs give a more accurate approximation.

Rediscretisation of the curve

As the fire perimeter expands then the distances between adjacent discretising points increases so introducing errors into the approximations of $x_s(s,t)$ and $y_s(s,t)$, especially in regions of high curvature. To solve this problem extra discretising points are added to the curve when the distances between adjacent points becomes large. There are many different and equally effective ways of doing this, with the technique used here being that developed by Richards, (1990).

If l_k is the length of line segment connecting the discretising points at $i = k$ and $i = k$-1, and α_k and α_{k-1} are the acute angles between the line segment and the next and previous line segments respectively, then if

$$max(\cos(\alpha_k/2),\cos(\alpha_{k-1}/2))>T/l_k \quad (11)$$

a new discretising point is added at the midpoint of the line segment, where T is a specified threshold value. This process is repeated recursively on both halves of the line segment, until the condition is satisfied by all the new points added.

Burnt regions

The numerical solution procedure is straight forward provided the functions F and G can be evaluated at each point and time on the fuel bed. Problems arise however when the fire has burnt a region of the fuel bed and then a separate region or even another fire attempts to burn into it. The detection of whether a region has been burnt, so that the values of functions F and G are now zero in that region, is a non trivial process computationally.

The algorithm described in the next section although not inexpensive was found to be reliable, and allows the simulation of highly complex fire scenarios. The algorithm is based upon detecting whether a discretising point has entered a region that is interior to the curve, i.e. a burnt region. The moment a discretising point enters a burnt region it is declared inactive, and remains stationary for the rest of the simulation.

Detection of Active and Inactive Discretising Points Using the Turning Number

Definition and calculation of the turning number

At time $j\Delta t$ the perimeter is approximated the set of discretising points $(x_{i,j},y_{i,j}), i = 0... n$, which together with the line segments joining consecutive points forms an orientated closed curve. If the curve has no horizontal line segments then each line segment can be considered to have an upward or a downward orientation. Figure 1 shows such a curve where the arrows indicate the direction associated with each line segment.

Provided the curve has no exactly horizontal line segments, when a horizontal line is drawn from a point in the plane that is external to the curve to a position sufficiently far to the left to also be external to the curve, then the number of intersections of this line with line segments of an upward orientation will equal those of a downward orientation, e.g. the top three dotted lines in Figure 2. If the point is not external to the curve then the difference between the number of intersections

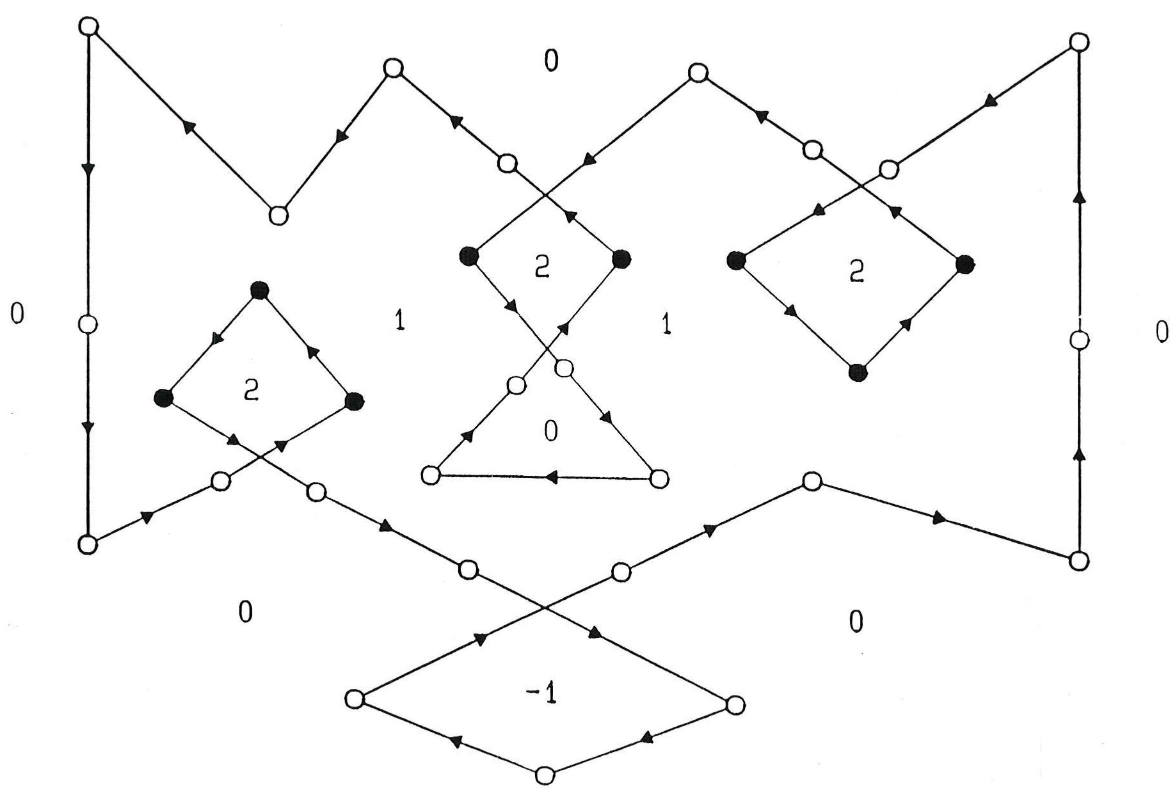

Figure 1. The fire perimeter represented as a closed directed curve, with the arrows indicating the direction associated with each line segment. The unfilled circles are the active discretising points and the solid circles are the inactive ones.

with line segments of an upward orientation and a downward orientation will be non zero, with the difference being called the turning number of the point relative to the curve, e.g. the bottom three dotted lines of Figure 2. The turning number of a point is the number of times a particle traversing a directed path around the curve will rotate counter clockwise around the point, (with clockwise rotations cancelling counter clockwise rotations), and will be zero if and only if the point is external to the curve.

The curve itself now partitions the plane into regions of equal turning number, with those regions external to the curve having a turning number of zero. A discretising point is now considered to be active if it is adjacent to a region of zero turning number. Figure 1 shows a curve together with the regions of equal turning number, with the unfilled circles being the active discretising points, and the solid circles being the inactive ones.

Provided a line segment from a different part of the curve does not pass through a discretising point, then the discretising point will be adjacent to two regions of different turning number. One of these regions can be considered to be to the left of the discretising point and the second region is either to the right, above or below. An algorithm to determine the turning number of these two regions is now presented.

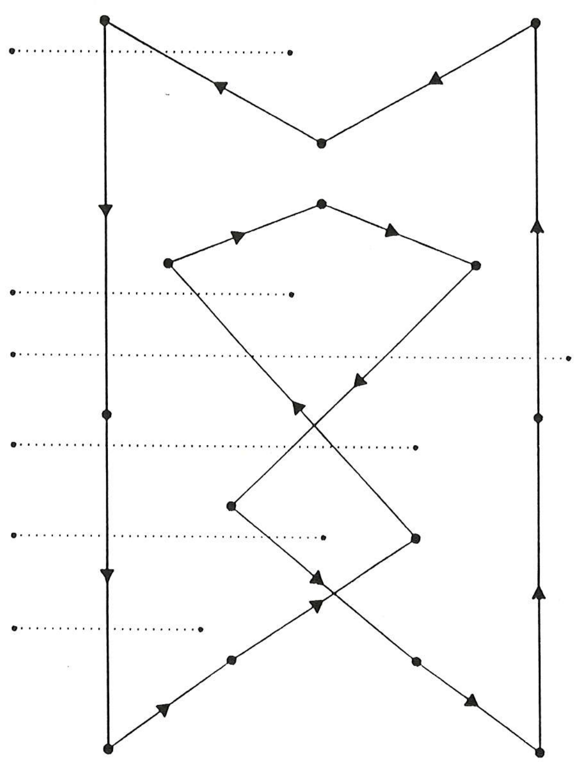

Figure 2. A closed directed curve where the dashed lines are horizontal lines drawn from points not on the curve, to positions sufficiently far to the left of the curve so as to be external to the curve. The arrows indicate the direction associated with each line segment.

Determination of the turning number of the left hand region

To determine the turning number of the region to the left of a discretising point, firstly a horizontal line is drawn from the discretising point to a position that is to the left of the whole curve, (and hence external to the curve). The difference between the number of intersections of this horizontal line with line segments of a downward orientation and an upward orientation is now the turning number of the left hand region. It should be noted that the intersection of the horizontal line with the curve at the actual discretising point in question is not included in this calculation.

In the event of this horizontal line passing exactly through another discretising point then the line is intersecting with two line segments at this point. If both line segments are of the same orientation a single intersection of this orientation is considered to have occurred. If the line segments are of a different orientation then the horizontal line is technically only touching the curve and no intersection is registered.

Having determined the turning number of the left hand region, if it is zero the discretising point is considered active and the process is complete, otherwise the turning number of the second region must be calculated.

Determination of the turning number of the second region

To calculate the turning number of the second region the contribution of the curve intersecting the horizontal line at the discretising point is also included in the difference between the number of upward and downward intersections.

If both the line segments adjacent to the discretising point are of the same orientation then the second region will be to the right of the first region, and a single intersection with the same orientation of the two line segments is considered to have occurred. If the two line segments adjacent to the discretising point are of different orientations then the second region will lie either above or below the point. This being the case then an intersection with an orientation equal to that of the left hand line segment is considered to have occurred. If both the adjacent discretising points are above the point in question then the left hand line segment is defined as the one that subtends the largest counterclockwise angle to the x-axis, otherwise it is the one that subtends the largest clockwise angle.

Having determined the turning number of the second region the process is now complete, with the discretising point being active if the turning number is 0.

Horizontal line segments and multiple adjacent regions

It is possible for a line segment to be exactly horizontal, so that the concept of an upward or a downward orientation is not well defined. The algorithm can be modified to account for this situation, however, it becomes unwieldly and more difficult to implement. To solve this problem the curve is scanned for horizontal line segments, if one is found then one of its end points is perturbed by a very small amount, so as to render the line segment non-horizontal. Even on a micro computer this perturbation can be made as small as 10^{-13} of a unit, and hence well within the error of the model and the numerical method.

Although rare, it is possible for a line segment from a separate part of the curve to pass exactly through the discretising point. If this is the case then the discretising point is adjacent to more than 2 regions of different turning number. Again it is possible to modify the algorithm to account for this situation, however due to the way in which fires burn even if the point is adjacent to region of zero turning number it is about to be instantaneously consumed by fire. The algorithm merely declares such points as inactive.

Multiple fires

When dealing with more than one fire each fire will be represented as a separate closed curve, and for a discretising point to be active two conditions must be satisfied. Firstly the discretising point must be active if the fire of which it is a part were the only one burning, which is determined by applying the algorithm in its basic form. Secondly the discretising point must be external to all of the other fires, which is determined by calculating its turning number relative to each of the other fires, which should all be zero for the discretising point to be active. The turning numbers are calculated in the previously described manner. If a discretising point lies exactly on the perimeter of another fire then it is declared inactive.

General comments

Although the literal description of the algorithm is fairly involved, it translates easily into computer code. This is not an inexpensive algorithm as it is of order n^2, where n is the number of discretising points. However, it is not as expensive as the order n^2 would suggest since it involves very few arithmetic operations. The algorithm has been defined in its most basic form, and the authors feel that there is much room for developing this idea further so as to improve its performance.

Example Simulations

The following simulations illustrate the capabilities of the simulation procedure for a number of non trivial fire scenarios. The point source ignitions are approximated by a small circle 0.01 of a spacial unit in radius, with 32 points evenly distributed around it. The times for the simulations are those for a 486 30MHz micro computer with a floating point processor, and are given to the nearest minute. It should be born in mind when evaluating these times that small time steps have been used to pick up the fine details of the fire behaviour, and for the purposes of illustration the simulation procedure has been presented in its most basic form. Even with present day workstation technology, times 40 times faster can be achieved.

Figure 3 demonstrates the capabilities of the simulation procedure for a fire situation involving a system of barriers and a wind direction change. For Figure 3A the fuel is such that for a constant wind direction the fire will expand as a double ellipse, with a blunt head and a sharp back fire region. The fire perimeters are displayed at equal intervals of time apart, with 20 intermediate perimeters being calculated between each displayed perimeter. Figure 3B is the same as Figure 3A save that the fuel is such that for a constant wind velocity the fire perimeter shape is a double ellipse with a sharp head and a blunt back fire region. The forward, flank and back spread rates are the same for both the double ellipses, in fact the fire shape in Figure 3B is that of Figure 3A turned the other way around. The resulting fire perimeters are certainly different, with the significance of the difference depending on what information is required from the simulation. A detailed discussion of the effects of the perimeter shape used is best left to a separate study, although the comment can be made that the head fire region of the sharp headed ellipse takes longer to move around a barrier, which is because of the more rapid decrease in the rate of spread around the perimeter as one moves away from the head fire region. The time for the each of the simulations was 17 minutes.

Figure 4 demonstates the capabilities of the simulation procedure for four interacting elliptical fires

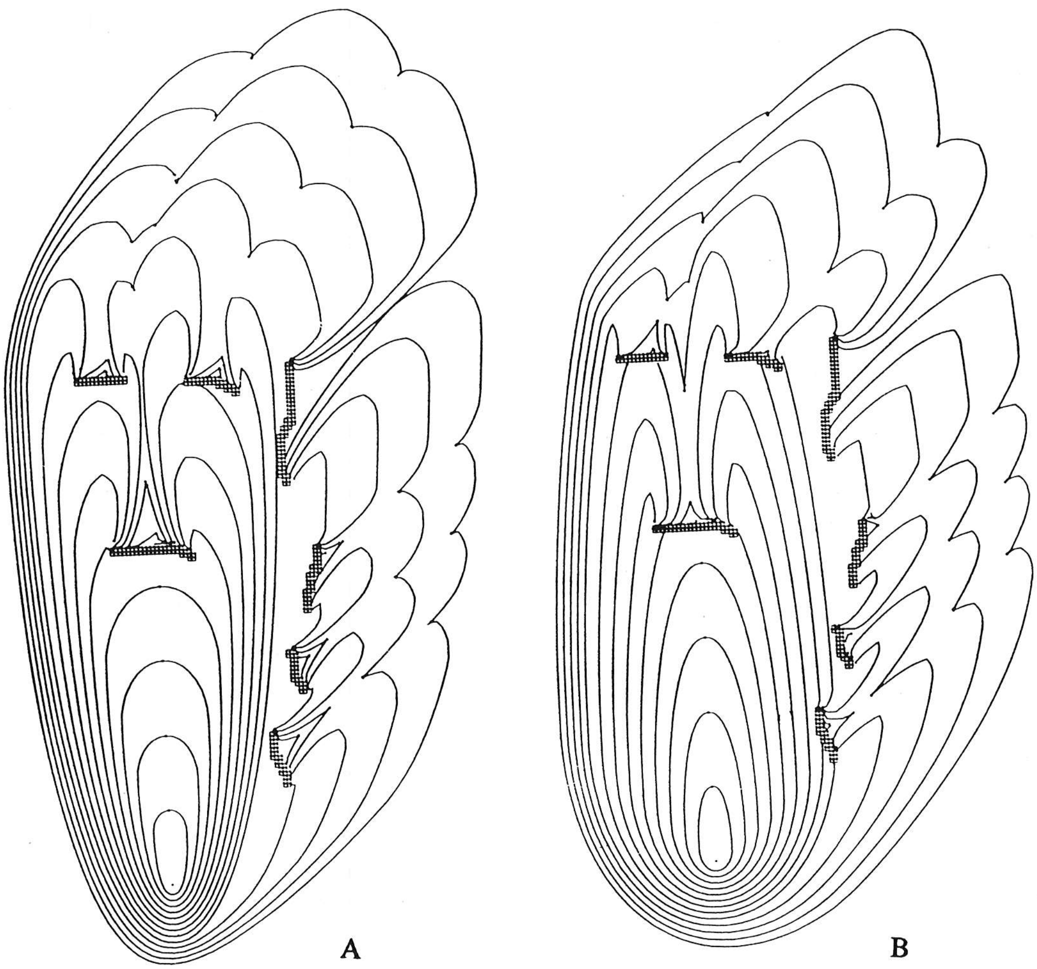

Figure 3. Double elliptical fires burning past a sequence of barriers, together with a wind direction change from $\theta = 0$ to $\theta = -\pi/4$ at $t = 80$ time units. The forward, flank and back spread rates for both fires are 1.85, 0.66, and 0.15 spacial units per unit of time respectively. The fire shape for a constant wind direction in 3B is the inverted version of that for 3A. The perimeters are displayed at intervals 10 units of time apart, with 20 intermediate perimeters being calculated, i.e. $\Delta t = 0.5$ of a time unit. A rediscretising threshold of $T = 1.5$ spacial units was used.

starting at different times, with a wind direction change over a fuel bed consisting of two fuel types and a system of barriers. The fire perimeters are displayed at equal intervals of time apart with 30 intermediate perimeters being calculated between each displayed perimeter. The separate fires eventually meet and form interfaces in a similar manner to the way in which the arms form by passing around a barrier. The dotted regions are areas of slower burning fuel. As the fire enters these regions it slows down whilst continuing along the outer edge of region with the faster spread rate. This causes the perimeter to bend and thus a head fire region separate from the main one forms. The borders of these regions are irregular in nature and as such cause sequences of very small head fires to form that burn into each other and eventually dissipate. This is responsible for the irregular nature of the fire perimeter after passing over a fuel interface and small time steps are required to pick up the details of the irregularity. The time for the simulation was 23 minutes.

Conclusions

A simulation procedure based on the solution of a pair of partial differential equations has been presented that can simulate fire spread for highly complex fire scenarios. The two main components of the procedure are the basic solution method for the solution of the differential equations, which uses Euler's method, and the detection of burn out that uses the turning number

Figure 4. Four elliptical fires, ignited at different times, burning over two different fuels types and a sequence of barriers, together with a wind direction change at $t = 40$ time units. In the dotted areas the forward, flank and back spread rates are 0.925, 0.33 and 0.075 spacial units per unit of time respectively, in the remaining faster burning fuel they are 1.85, 0.66 and 0.15 respectively. The fire at the center bottom was ignited at $t = 0$ time units with the two fires to the left and right ignited at $t = 20$ time units; the top right fire was ignited at $t = 55$ time units. The fire perimeters are displayed 10 time units apart, with 30 intermediate perimeters being calculated, i.e. $\Delta t = 0.333$ time units. A rediscretising threshold $T = 1.5$ spacial units was used.

of a point relative to the perimeter. Euler's method was used for purposes of illustration and there are many more sophisticated methods of solution that can be used, with further work being required to determine the most appropriate one. The algorithm for detecting burnt regions was found to be reliable and the authors feel that there is much room for further optimisation.

Despite the basic form in which the procedure has been presented, it allows for the simulation of fire scenarios of a complexity that previously has not been done.

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